

PMATH 347 Assignment 1 Fall 2016

This assignment is due at the beginning of the class on Wednesday, September 21.

Question 1

Let

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix} \quad \text{and} \quad \tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}.$$

In each case, solve for $\chi \in S_4$.

- (a) $\chi\tau\sigma = \varepsilon$.
- (b) $\tau\chi\sigma^{-1} = \sigma$.

Question 2

Let S be a non-empty set and let $M = \{X : X \subseteq S\}$ denote the set of all subsets of S . Let \cup and \cap denote the usual set union and set intersection.

- (a) Determine if (M, \cup) is a monoid or not. Justify your answer.
- (b) Determine if (M, \cap) is a monoid or not. Justify your answer.
- (c) Determine if the empty set is a monoid. Justify your answer.

Question 3

Let G be a group and $a, b \in G$.

- (a) Suppose that $a^6 = 1$ and $ab = ba^2$. Prove that $a^3 = 1$ and $aba = b$.
- (b) Suppose that for $n \in \mathbb{N} \cup \{0\}$, we have $(ab)^n = 1$. Prove that $(ba)^n = 1$.
- (c) Extend the statement in (b) to all $n \in \mathbb{Z}$.

Question 4

(a) Prove that up to isomorphism, there are only two groups of order 4, the cyclic group C_4 and a noncyclic group K_4 , whose Cayley table is shown below.

| K_4 | 1 | a | b | c |
|-------|-----|-----|-----|-----|
| 1 | 1 | a | b | c |
| a | a | 1 | c | b |
| b | b | c | 1 | a |
| c | c | b | a | 1 |

The group K_4 is called the *Klein 4 group*.

- (b) Prove that $K_4 \cong C_2 \times C_2$.

Question 5

- (a) Prove that a group G is abelian if and only if $(gh)^{-1} = g^{-1}h^{-1}$ for all $g, h \in G$.
- (b) Prove that a group G is abelian if $g^2 = 1$ for all $g \in G$.
- (c) Determine if the converse of (2) is true or not. Justify your answer.