

# PMATH 347 Assignment 1 Fall 2016

This assignment is due at the beginning of the class on Wednesday, September 21.

## Question 1

Let

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix} \quad \text{and} \quad \tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}.$$

In each case, solve for  $\chi \in S_4$ .

- (a)  $\chi\tau\sigma = \varepsilon$ .
- (b)  $\tau\chi\sigma^{-1} = \sigma$ .

## Question 2

Let  $S$  be a non-empty set and let  $M = \{X : X \subseteq S\}$  denote the set of all subsets of  $S$ . Let  $\cup$  and  $\cap$  denote the usual set union and set intersection.

- (a) Determine if  $(M, \cup)$  is a monoid or not. Justify your answer.
- (b) Determine if  $(M, \cap)$  is a monoid or not. Justify your answer.
- (c) Determine if the empty set is a monoid. Justify your answer.

## Question 3

Let  $G$  be a group and  $a, b \in G$ .

- (a) Suppose that  $a^6 = 1$  and  $ab = ba^2$ . Prove that  $a^3 = 1$  and  $aba = b$ .
- (b) Suppose that for  $n \in \mathbb{N} \cup \{0\}$ , we have  $(ab)^n = 1$ . Prove that  $(ba)^n = 1$ .
- (c) Extend the statement in (b) to all  $n \in \mathbb{Z}$ .

## Question 4

(a) Prove that up to isomorphism, there are only two groups of order 4, the cyclic group  $C_4$  and a noncyclic group  $K_4$ , whose Cayley table is shown below.

$K_4$	1	$a$	$b$	$c$
1	1	$a$	$b$	$c$
$a$	$a$	1	$c$	$b$
$b$	$b$	$c$	1	$a$
$c$	$c$	$b$	$a$	1

The group  $K_4$  is called the *Klein 4 group*.

- (b) Prove that  $K_4 \cong C_2 \times C_2$ .

## Question 5

- (a) Prove that a group  $G$  is abelian if and only if  $(gh)^{-1} = g^{-1}h^{-1}$  for all  $g, h \in G$ .
- (b) Prove that a group  $G$  is abelian if  $g^2 = 1$  for all  $g \in G$ .
- (c) Determine if the converse of (2) is true or not. Justify your answer.